A Universal Overlay for Surface Impedance Calculations for Composite Conductors

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Abstract—**Surface impedance calculations for composite conducting surfaces made of two different metals can be made amenable to a universal graphical solution. This is in the form of an overlay to be superimposed on the reflection coefficient plane and is therefore useful in conjunction with either the Smith Chart or the "$Z-\theta$" Chart. An example is given of a composite conductor consisting of a thin nickel interfacing layer underlying a thick layer of gold, such as might be found in the construction of microstrip circuit elements.**

I. Introduction

The reflection coefficient plane, of which the Smith Chart is an example, is especially appropriate for portraying, as well as for providing, solutions to a variety of electromagnetic wave problems. It is particularly helpful for those cases involving multiple reflections.

The composite conducting surface used when forming certain types of microstrip circuit elements and transmission lines arises, for example, when gold is applied to ceramic dielectrics using an intermediate bonding metal such as nickel. Although the electromagnetic wave within the dielectric faces a nickel surface, the layer is so thin that the wave reflection process is still dominated by the overlying relatively thick gold layer. This configuration is illustrated in Fig. 1.

The surface impedance presented by the composite conductor to the wave at the dielectric boundary is readily determined by conventional analysis [1],[2]; however that solution involves several steps of complex arithmetic, whereas a single reflection coefficient plane overlay can be developed to portray a universal solution for any bimetallic layer. This overlay can be superimposed on either the Smith Chart or on the "$Z-\theta$" Chart, both of which are themselves reflection coefficient plane overlays.

II. Theory

If a bonding conductor of thickness t and intrinsic impedance η_b separates a dielectric region from a thick external layer of intrinsic impedance η_e, the surface impedance presented to the electromagnetic wave at the dielectric boundary is [3]:

$$Z_s = \eta_b \frac{1+\Gamma e^{-2\gamma t}}{1-\Gamma e^{-2\gamma t}}. \tag{1}$$

In this expression Γ is the reflection coefficient at the interface between the two conductors and is given by

$$\Gamma = \frac{(\eta_e/\eta_b)-1}{(\eta_e/\eta_b)+1}. \tag{2}$$

The propagation constant γ is the value in the bonding layer and is generally expressed in terms of the skin depth δ as

$$\gamma = \frac{1}{\delta} + j\frac{1}{\delta}. \tag{3}$$

The intrinsic impedance of a metal is well known as

$$\eta = \sqrt{j\omega\mu/\sigma}$$

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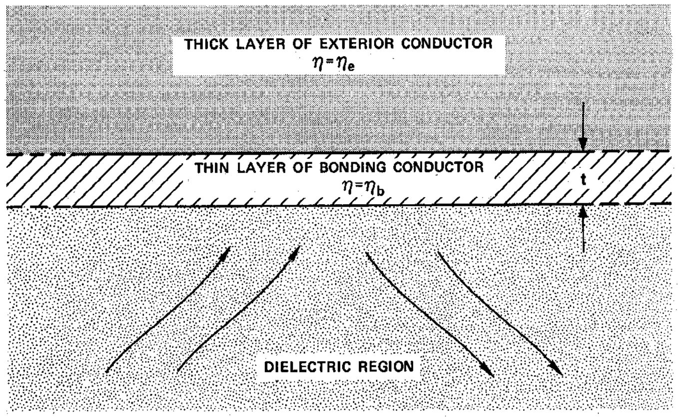


Fig. 1. Composite conductor.

hence

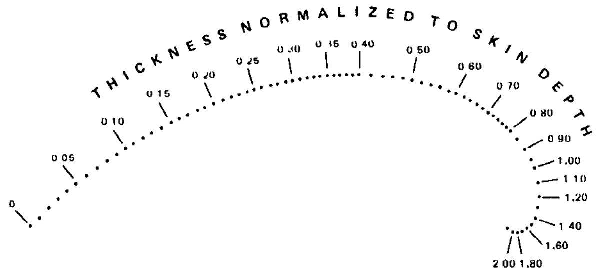
$$\eta_e/\eta_b = \sqrt{(\mu_e/\mu_b)(\sigma_b/\sigma_e)}$$

and is real so that Γ is real although η has a phase of 45°. In these expressions μ represents the permeability and σ the conductivity in the appropriate region. The transformation of (1) is generally portrayed as a locus in the plane of the complex variable $w=\Gamma e^{-2\gamma t}$. Corresponding values of the normalized impedance $\zeta = Z_s/\eta_b$ can be identified either on the Smith Chart or on a "$Z-\theta$" Chart in which an impedance is expressed in terms of its magnitude and phase. The latter chart is preferable since the final determination of Z_s requires that ζ be multiplied by η_b which has a phase of 45°. The locus of w where γ has the form of (3) is a spiral making an angle of 45° with the radius vector. As $t \to \infty$ the spiral terminates at $w=0$, the center of the Chart. The initial value of $w=\Gamma$ is defined as $\Gamma = e^{-2t_0/\delta}e^{j\theta_0}$. Thus transformation through the layer gives $w = e^{-2(t+t_0)/\delta}e^{-j2t/\delta}e^{j\theta_0}$ which may be written as $w = e^{-2(t+t_0)/\delta}e^{-j2(t+t_0)/\delta}e^{j2t_0/\delta}e^{j\theta_0}$, the first two terms of which are the universal overlay curve. The latter equation simply states that the parameter $(t+t_0)/\delta$ is to be identified on the overlay curve which is to be given, in addition to θ_0, a further initial rotation in the amount of $2t_0/\delta$-*a* rotation which is required to force the overlay to pass through $w=\Gamma$ at $t=0$. Since the value of t_0 is of no physical importance there is no actual need to adjust the overlay so that $t_0=0$ coincides with a radius of unity in the w plane. In fact it is this aspect of the overlay which makes it universal, since it need not be redrawn if a different size of Smith Chart (or "$Z-\theta$" Chart) is used. Different chart sizes simply produce different values of t_0/δ. The value of the normalized surface impedance at the dielectric boundary is read through the overlay by adding t/δ to the starting value of the parameter t_0/δ. The fact that t_0 is a constant in each case is shown in the Appendix.

The spiral $w=e^{-2\tau}e^{-j2\tau}$ has been prepared in Fig. 2. Incremental values of τ are identified on the spiral and are spaced closely enough so that it can be used for graphical calculation. The end point, $\tau \to \infty$, which is required for centering, is not labelled. The spiral may be of any size that is convenient, and for actual use must, of course, be prepared as a transparency.

III. Example

A nickel ($\sigma = 1.42\times10^7$ mhos/m, $\mu = 110\mu_0$) layer of thickness 200 Å is used to bond a relatively thick layer of gold ($\sigma = 4.54\times 10^7$ mho/m, $\mu=\mu_0$) to a ceramic dielectric. Operation is at a frequency of 2.46 GHz.



REFLECTION COEFFICIENT PLANE OVERLAY
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Fig. 2. Universal overlay.

To find the surface impedance Z_s the first step is the evaluation of the gold surface impedance normalized to the nickel, i.e., $\eta_e/\eta_b = \sqrt{(1/110)(1.42/4.54)} = 0.0534$. This normalized impedance must be transformed through the nickel layer which is characterized in terms of its skin depth of $1/\sqrt{\pi f \mu \sigma} = 1/\sqrt{(\pi)(2.46\cdot 10^9)(110)(4\pi 10^{-7})(1.42\cdot 10^7)} = 0.2567\cdot 10^{-6}$ m. The layer of 200 Å is thus 0.0779 skin depths in thickness.

The spiral convergence point of the overlay is centered in the reflection coefficient plane and the spiral rotated until it passes through the original point Γ, which is the example is at $(0.0534-1)/(0.0534+1) = -0.899$. The latter calculation is, of course, generally dispensed with; it is redundant since the Smith Chart (or "$Z-\theta$" Chart) locates the point from the normalized impedance value directly. The size of overlay used for this example was observed to have a parameter value of t_0/δ of 0.202 when fitted to the initial $\zeta_0 = 0.0534$. The addition of $t/\delta = 0.0779$ to the starting value of the parameter gives $(t+t_0)/\delta = 0.280$ which overlaid a normalized impedance of $0.131 + j0.077$. Since $\eta_b = 0.274 + j0.274\ \Omega$ the actual surface impedance is $Z_s = 0.015 + j0.057\ \Omega$. It may be noted that in this example the effect of the thin bonding layer is primarily on the reactive component of the impedance, since for gold $\eta_e = 0.014 + j0.014\ \Omega$.

IV. Appendix

An exponential spiral is described in polar coordinates r and θ and characterized in terms of the parameter t_1 as

$$r = K_1 e^{-2t_1/\delta} \qquad \theta = -2t_1/\delta + \theta_1.$$

A second spiral is characterized in terms of the parameter t_2 as

$$r = K_2 e^{-2t_2/\delta} \qquad \theta = -2t_2/\delta + \theta_2.$$

When the parameter changes by a small amount, in the first case

$$dr = -(2/\delta) r\, dt_1 \qquad d\theta = -(2/\delta)\, dt_1$$

and in the second

$$dr = -(2/\delta) r\, dt_2 \qquad d\theta = -(2/\delta)\, dt_2.$$

If the spirals pass through the same point r, θ the same increment in the parameters, $dt_1 = dt_2$ is required to produce the same increment in r and θ in either case. Hence the coincidence of every pair of points on the superimposed spirals requires that $t_1 = t_2 + t_0$, where t_0 is an arbitrary constant if K_1, K_2, θ_1, and θ_2 are arbitrary.

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A Time Domain Reflectometer Using a Semiautomatic Network Analyzer and the Fast Fourier Transform

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***Abstract*—A time domain reflectometer system is simulated by measuring the reflection coefficient in the frequency domain and then computing the time domain signal by the Fourier transform. The program has been written for the Hewlett-Packard 8409A Semiautomatic Network Analyzer. The computation time has been minimized by using the fast Fourier transform. The problems imposed by the difficulty of switching the HP 8409A between low- and high-frequency ranges are also discussed.**

I. Introduction

One advantage of the computer controlled automatic network analyzer is that the measured results can easily be used for further calculations. A very interesting application is to simulate a time domain reflectometer by means of the Fourier transform. An article [1] which described a system using the Hewlett-Packard 8542B Automatic Network Analyzer was published in 1974. The Fourier transform was performed by a truncated Fourier series.

A few years ago Hewlett-Packard introduced the HP 8409A Semiautomatic Network Analyzer which is a low cost version of the 8542B. The present article is a description of a Fourier transform program developed for the HP 8409A controller, which is a HP 9825 desktop computer. The program is different in several aspects from the program in [1]. The computation is performed by a fast Fourier transform implementation of the discrete Fourier transform, which is considerably faster than using a truncated Fourier series [2].

In the HP 8409A system the switching between the low frequency range (0.1–2 GHz) and the high-frequency range (2–18 GHz) is performed by changing some instruments, which is a rather time consuming operation. Therefore, only the high frequency range is used in the program.

II. Implementation

The Fourier transform of a signal consisting of a dc signal, a basic frequency and harmonics of the basic frequency, all with equal amplitude, is a pulse function. If the signal is band limited

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